BARREL

A modern backend for Atelier B in Lean

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We can make it safer with a modern proof assistant.

Context

The B method and Atelier B

- **The B method**: formal software development via refinement of state machines.
- **Atelier B**: industrial tool for B, including
  - a PO generator,
  - internal/external (automated) provers,
  - an interactive prover.

Context

Why a modern backend?

- Atelier B's prover is **trust-by-tradition** (and closed).
- Existing alternatives are partial:
  - Isabelle/HOL for Event-B (Rodin plugin, DSL)
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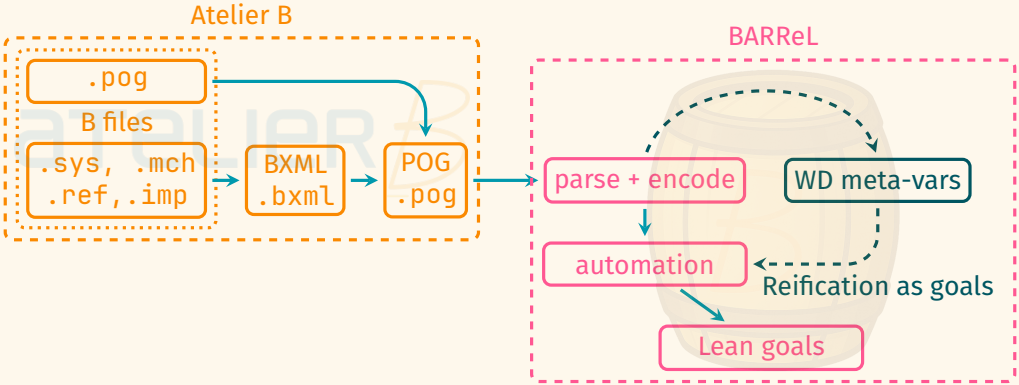
  - Rocq for B semantics (no integrated backend)
  - SMT-based discharge of POs (BEer, ppTrans)
- We want:
  - interactive proofs on the original B POs (or close enough),
  - a rich mathematical library (*e. g.*, Mathlib),
  - and by-construction treatment of WD.

B Automated tRanslation for Reasoning in Lean

BARReL
Pipeline



Encoding B in Lean

Sets, relations, functions

- B notations are reproduced verbatim: $\subseteq$, $\in$, $\cup$, $\cap$, $\mathcal{P}$, FIN, FIN_1, $\rightarrow$, $\nrightarrow$, $\rightarrowtail$, ...

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- Encoding: straightforward set-based mapping $\mathcal{B} \rightarrow$ LEAN using Mathlib.
- Coverage: logic, sets, relations, functions, arithmetic, sequences, ...

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structure min.WD [PartialOrder α] (S : Set α)
  where
    isBoundedBelow :
      ∃ x ∈ S, ∀ y ∈ S, x ≤ y

def min (S : Set α) (wd : min.WD S) : α :=
  Classical.choose wd.isBoundedBelow
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```
structure app.WD (f : SetRel α β) (x : α)
  where
    isPartialFunction : f ∈ dom f → ran f
    isInDomain        : x ∈ dom f

def app (f : SetRel α β) (x : α)
  (wd : app.WD f x) : β :=
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Impossible to build a term using a partial operator without supplying a WD proof.

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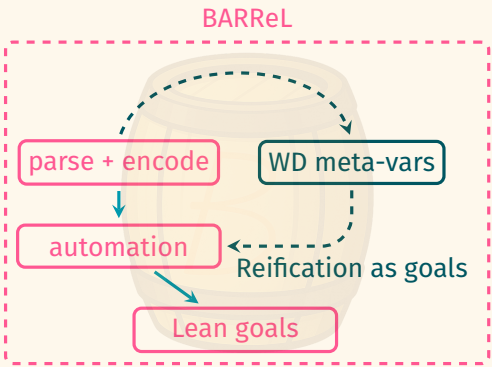
WD as metavariables, reified as goals

When BARReL encodes a B formula containing a partial operator:



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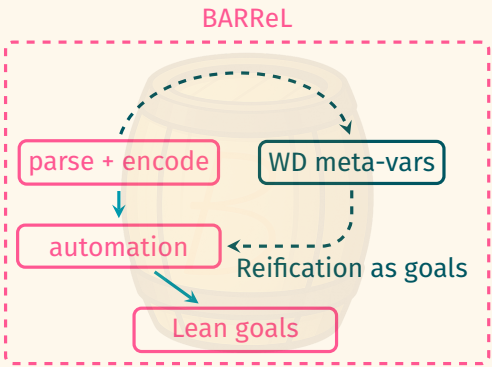

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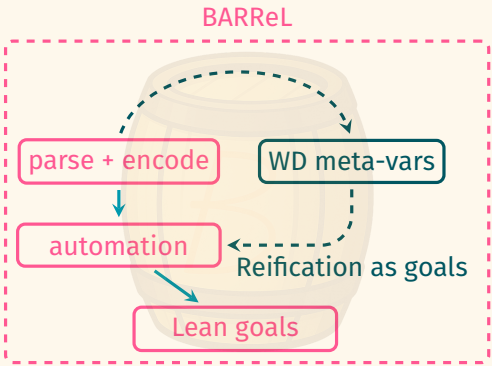

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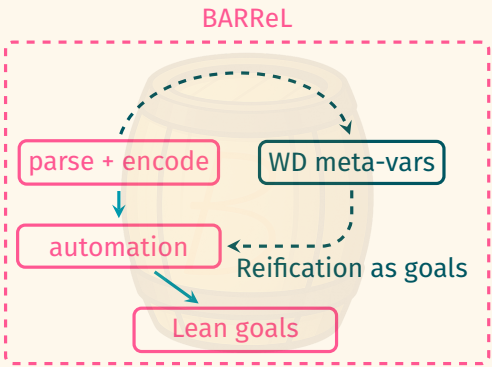

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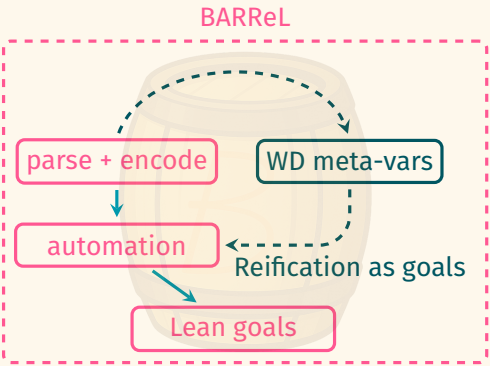

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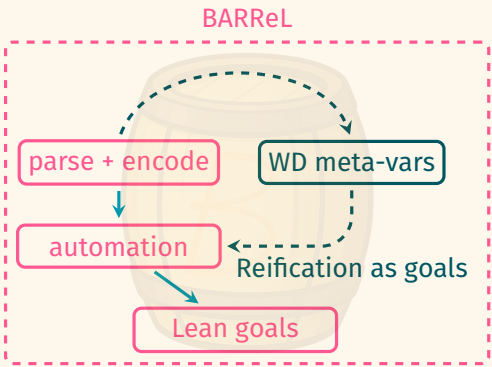

$\Longrightarrow$ **no PO step can quietly rely on an ill-defined term.**

Encoding B in Lean

Automation

WD tactics and categorization for WD lemmas:

Tactic	WD property
`wd_min`	bounded below + non-empty
`wd_max`	bounded above + non-empty
`wd_app`	functional + in domain
`wd_card`	finite

- Get rid of elementary set-theoretic facts.
- Non-trivial facts are left to the user, with full Mathlib at hand.

Encoding B in Lean

Automation

```
@[wd_min]
theorem min.WD_of_finite [LinearOrder α] {S A : Set α} (h : S ∈ FIN₁ A) :
  min.WD S

@[wd_app]
theorem app.WD_of_overload {A C : Set α} {B D : Set β} {f g : SetRel α β} {x : α}
  (hf : f ∈ A ⇸ B) (hg : g ∈ C ⇸ D) (hx : x ∈ dom f ∨ x ∈ dom g) :
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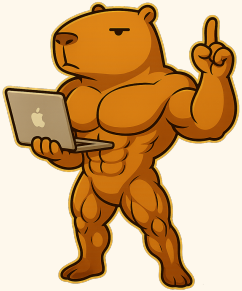
User-extensible: any new lemma tagged `@[wd_*]` is picked up automatically.

Demo

Live!



Demo time!

Conclusion

What we have, where we're going

- BARReL is lightweight and modular (≈ 1.3*k* LoC impl., ≈ 1.3*k* LoC lib.),
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Future work, ordered by difficulty:

- Cover more of the B language,
- Beef up automation,
- Reduce WD redundancy and improve scalability,
- Implement a **native**, **verified** PO generator.