# Enhancing B language reasoners with SMT techniques

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# Outline

1. **Introduction**
   - Ph.D. subject
   - B and Atelier B
   - SMT-LIB
2. **Encoding B proof obligations in SMT-LIB using HOL**
   - Encoding sets
   - Encoding functions
   - Example
3. **Conclusion**

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**Formal methods** for safety-critical systems, e.g. railways

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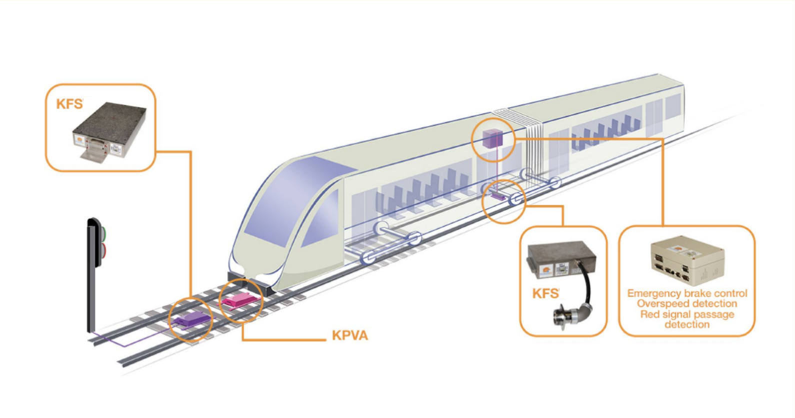

**Subject**: Enhancing B language reasoners with SMT techniques

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(declare-fun f (→ Int (Option Int)))
(assert (forall ((x Int))
  (= (not (= (f x) none)) (⩽ 0 x)))))
(assert (forall ((x Int)) (⇒
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VARIABLES
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INITIALISATION
  a : INTEGER & b :
   INTEGER &
  f : NATURAL --> a .. b
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# B and Atelier B

### B

- Formal method for software and hardware development
- Based on **ZFC** + Predicate Logic
- Structured around abstract machines, variables, invariants, and operations.

### Atelier B

- Suite of tools for B development
- Includes a proof obligation generator and a (FO) predicate prover
- Tries to automatically discharge proof obligations

## SMT-LIB

- Standard input format for SMT solvers (e.g. z3, cvc5, veriT)
- Based on many-sorted first-order logic
- Comes with many theories (e.g. arrays, integer and real linear arithmetic)

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## Current encoding (ppTrans)

A **first-order** encoding of sets:

- Based on FOL with equality and uninterpreted functions (EUF)
- Sets are only *specified* through the use of an uninterpreted predicate $\in$
- Only expressions like $x \in S$ are encoded

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(declare-fun S () (P Int))
(declare-fun e1 () Int)
(declare-fun e2 () Int)
(declare-fun e3 () Int)
(declare-fun ∈0 ((Int) (P Int)) Bool)

(assert (forall ((x Int)) (=
  (∈0 x S)
  (or (= x e1) (= x e2) (= x e3)))))
```

## Suggested encoding

A **higher-order** encoding of sets:

- Uses some extensions of SMT-LIB 2.6 to HOL brought by cvc5 ($\lambda$-abstraction, arrow type constructor)
- Sets are represented by their characteristic predicate: no need for membership predicate!

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(declare-const e1 Int)
(declare-const e2 Int)
(declare-const e3 Int)
(define-const S (-> Int Bool) (lambda
    ((x Int))
  (or (= x e1) (= x e2) (= x e3))))
```

Currently, functions such as $f : A \nrightarrow B$ are represented by:

- the set $\mathcal{P}(A \times B)$
- axiom stating that the relation is functional

$$\forall x\, y\, z.\, x \mapsto y \in f \land x \mapsto z \in f \Rightarrow y = z \quad \text{(functionality)}$$

- additional axioms accounting for properties of $f$ (totality, partiality, injectivity...)

$$\forall x_1\, y_1\, x_2\, y_2.\, x_1 \mapsto y_1 \in f \land x_2 \mapsto y_2 \in f \land y_1 = y_2 \Rightarrow x_1 = x_2 \quad \text{(injectivity)}$$

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\exists\, N : \iota, f : \tau \to \iota \cdot\ & \\
& \forall x : \tau, y : \tau, z : \iota \cdot f(x) = z \land f(y) = z \Rightarrow x = y \qquad \land \\
& \forall x : \tau \cdot x \in S \Rightarrow f(x) < N
\end{aligned}
$$

# A non-working example

Let $f \in \mathbb{Z} \nrightarrow \mathbb{Z}$. Let op be the following operation:

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op (x, y) =
PRE
  x : INTEGER & y : INTEGER // x ∈ ℤ ∧ y ∈ ℤ
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  f := f \/ {x |-> y}       // f := f ∪ {x ↦ y}
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- After $\mathrm{op}(0,2)$, $f = \{0 \mapsto 1, 1 \mapsto 0, 0 \mapsto 2\}$ ✗ (no longer a function)

## Idea of the encoding for ”true” functions:

$\cdot^{?}$ is a (post-fixed) low-priority notation for the `Option` type.
$\uparrow\cdot$ is a (pre-fixed) high-priority notation for the set-lifting operation defined inductively as follows:

- $\uparrow(A \times B) = \uparrow A \times \uparrow B$
- $\uparrow\mathcal{P}(A) = \mathcal{P}(\uparrow A)$
- $\uparrow\{x \in A \mid P\} = \uparrow A$
- $\uparrow\mathtt{Bool} = \mathtt{Bool}$
- $\uparrow\_ = \mathbb{Z}$ (e.g. $\uparrow\mathbb{N} = \uparrow\{e_i\}_{i\in\mathcal{I}} = \mathbb{Z}$)

Idea of the encoding for "true" functions:

## Rule: partial function

$f : A \nrightarrow B$ is encoded as:

- $f \in \uparrow A \to \uparrow B^?$
- $\forall x \in \uparrow A.\ f\ x \neq \mathtt{none} \Rightarrow x \in A$
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Idea of the encoding for "true" functions:

**Rule: total function**

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$$f : \mathbb{N} \to a..b \quad \hookrightarrow$$

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$$f : \mathbb{N} \to a..b \quad \hookrightarrow$$

$$\begin{cases} f \in \mathbb{Z} \to \mathbb{Z}^{?} \\ \forall x \in \mathbb{Z}.\, f\ x \neq \mathsf{none} \Leftrightarrow 0 \leq x \\ \forall x \in \mathbb{Z}.\, f\ x \neq \mathsf{none} \Rightarrow \exists y \in a..b.\, f\ x = \mathsf{some}\ y \end{cases}$$

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(assert (forall ((x Int)) (= (not (= (f x) none)) (⩽ 0 x))))
(assert (forall ((x Int)) (⇒ (not (= (f x) none))
  (exists ((y Int)) (and (⩽ a y) (⩽ y b) (= (f x) (some y)))))
   ))
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- Leverage recent extensions to fragments of **HOL** in SMT-LIB to encode B proof obligations
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## Questions?